Anatomy of Toroidal Drift Modes

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Toroidal Drift Mode Anatomy (Part I)

Can describe Toroidal ITG, and all three varieties of TEM modes with one dispersion relation.

Ions in Fluid Limit: $\omega_{Di}, k_{\|} v_{thi} \ll \omega$

$$\frac{\delta n_i}{n} = -\frac{e\delta\phi}{T_i}\left\{\frac{\omega_{*i}-\omega_{Di}}{\omega} + \frac{\omega_{*i}(1+\eta_i)\omega_{Di}}{\omega^2} + \left(b_i - \frac{k_{\|}^2 {v_{thi}}^2}{\omega^2}\right)\left(1 - \frac{\omega_{*i}(1+\eta_i)}{\omega}\right)\right\}$$

Circulating electrons with $\omega/k_{\|} \ll v_{the}$: adiabatic response

$$\frac{\delta n_{eC}}{n_{eC}} = \frac{e\delta\phi}{T_e}$$

Trapped electrons with $v_{the\|} \ll \omega/k_{\|} \ll v_{the}$

(feel curvature, ExB drifts, but no Landau damping)

$$\partial_t \delta n_{eT} + \nabla\cdot\left(n_{eT}\underset{\text{ExB drift}}{\delta\mathbf{V}_{eT}} + \underset{\text{curvature drift}}{\mathbf{V}_{eT}}\delta n_{eT}\right) = 0$$

$$\frac{\delta n_{eT}}{n_{eT}} = \frac{\omega_{*e}}{\omega - \omega_{De}}\frac{e\delta\phi}{T_e}$$

$\eta_i = d\ln T_i / d\ln n$

$\omega_D = k_\theta g/\Omega_c$

$g = v_{th}^2/R$

$b_i = k_\theta^2 \rho_i^2/2$

$\omega_* = k_\theta D_B / r_n$

$D_B = cT/ZeB$

$r_n^{-1} = -d\ln n/dr$

$\varepsilon_n = r_n/R_0$

$\tau = T_e/T_i$

Impose quasineutrality to obtain dispersion relation

$$\delta n_i = \delta n_{eC} + \delta n_{eT}$$

Refs:

B. Coppi and G. Rewoldt, PRL 33 (1974) 1329.

J.C. Adam, W. M. Tang, P.H. Rutherford, Phys. Fluids 19 (1976) 561.

C. S. Lui, M.N. Rosenbluth, W.M. Tang, Phys. Fluids 19 (1976) 1040.

B. Coppi and F. Pegoraro, Nuc. Fus. 17 (1977) 969.

W. M. Tang, G. Rewoldt, Liu Chen, Phys. Fluids 29 (1986) 3715.

Y.-K. Pu, PhD Thesis, MIT (1988).

B. Coppi, S. Migliuolo, Y.-K. Pu. Phys. Fluids B 2 (1990) 2322.

W.M. Daughton, PhD Thesis, MIT (1998).

Toroidal Drift Mode Anatomy (Part II)

First, improve on electron response with approximate kinetic theory...

$$\delta f_e \equiv \frac{e\delta\phi}{T_e} F_{me} + \delta h_e$$

$$\left(\omega - \omega_{De} + i\nu_{\mathrm{eff}} + i\frac{v_\parallel}{qR}\frac{\partial}{\partial\theta}\right)\delta h_e = -\frac{e\delta\phi}{T_e} F_{me}(\omega - \omega_{*e})$$

$$\oint \frac{d\theta}{v_\parallel}(\cdots) \Rightarrow \delta h_e \simeq -\frac{e\delta\phi}{T_e} F_{me} \frac{\omega - \omega_{*e}(1+\eta_e(E/T_e - 3/2))}{\omega - \omega_{De} + i\nu_{\mathrm{eff}}}$$

Integral over velocity space gives perturbed density:

$$\frac{\delta n_{eC}}{n_{eC}} = \frac{e\delta\phi}{T_e} \qquad \langle\cdots\rangle = \frac{2}{\sqrt{\pi}}\int_0^\infty dx\,\sqrt{x}e^{-x}(\cdots)$$

$$\frac{\delta n_{eT}}{n_{eT}} = \frac{e\delta\phi}{T_e}\left(1 - \left\langle\frac{\omega - \omega_{*e}(1+\eta_e(E/T_e - 3/2))}{\omega - \omega_{De} + i\nu_{\mathrm{eff}}}\right\rangle_E\right)$$

$$\boxed{\frac{\delta n_e}{n_e} = \frac{e\delta\phi}{T_e}\left(1 - \frac{n_{eT}}{n_e}\left\langle\frac{\omega - \omega_{*e}(1+\eta_e(E/T_e - 3/2))}{\omega - \omega_{De} + i\nu_{\mathrm{eff}}}\right\rangle_E\right)}$$

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Toroidal Drift Mode Anatomy: Resonant Modes

Construct Dispersion Relation:

$$\boxed{\begin{aligned}&\frac{\omega_{*i}-\omega_{Di}}{\omega}+\frac{\omega_{*i}(1+\eta_i)\omega_{Di}}{\omega^2}+\left(b_i-\frac{k_\parallel^2 v_{\mathrm{thi}}{}^2}{\omega^2}\right)\left(1-\frac{\omega_{*i}(1+\eta_i)}{\omega}\right)\\&+\frac{1}{\tau}-\frac{1}{\tau}\frac{n_{eT}}{n}\left\langle\frac{\omega-\omega_{*e}(1+\eta_e(E/T_e-3/2))}{\omega-\omega_{De}+i\nu_{\mathrm{eff}}}\right\rangle_E=0\end{aligned}}$$

1) Dissipative Trapped Electron Mode - destabilized by collisions

$$\begin{aligned}&\frac{\omega_{*i}-\cancel{\omega_{Di}}}{\omega}+\cancel{\frac{\omega_{*i}(1+\eta_i)\omega_{Di}}{\omega^2}}+\left(b_i-\cancel{\frac{k_\parallel^2 v_{\mathrm{thi}}{}^2}{\omega^2}}\right)\left(1-\cancel{\frac{\omega_{*i}(1+\eta_i)}{\omega}}\right)\\&+\frac{1}{\tau}-\frac{1}{\tau}\frac{n_{eT}}{n}\left\langle\frac{\omega-\omega_{*e}(1+\eta_e(E/T_e-3/2))}{\omega-\cancel{\omega_{De}}+i\nu_{\mathrm{eff}}}\right\rangle_E=0\end{aligned}$$

$$\boxed{\gamma_{DTEM}\propto\frac{\varepsilon^{3/2}\omega_{*e}^2}{\nu_{ei}}\left(\frac{3}{2}\eta_e+b_s\right)}$$

2) Collisionless Trapped Electron Mode - destabilized by resonant denominator

$$\begin{aligned}&\frac{\omega_{*i}-\cancel{\omega_{Di}}}{\omega}+\cancel{\frac{\omega_{*i}(1+\eta_i)\omega_{Di}}{\omega^2}}+\left(b_i-\cancel{\frac{k_\parallel^2 v_{\mathrm{thi}}{}^2}{\omega^2}}\right)\left(1-\cancel{\frac{\omega_{*i}(1+\eta_i)}{\omega}}\right)\\&+\frac{1}{\tau}-\frac{1}{\tau}\frac{n_{eT}}{n}\left\langle\frac{\omega-\omega_{*e}(1+\cancel{\eta_e}(E/T_e-3/2))}{\omega-\omega_{De}+i\cancel{\nu_{\mathrm{eff}}}}\right\rangle_E=0\end{aligned}$$

$$\boxed{\gamma_{CTEM}\simeq\omega_{*e}\left(\frac{2\pi r}{R}\right)^{1/2}\eta_e\left(\frac{R}{r_n}\right)^{3/2}\left(\frac{R}{r_n}-\frac{3}{2}\right)\exp(-R/r_n)}$$

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Toroidal Drift Mode Anatomy: Non-Resonant "Fluid" Modes

Expand electron response for $\omega_{De} \ll \omega$ ⇒ $$\frac{\omega-\omega_{*T}}{\omega-\omega_D} \simeq \left(1-\frac{\omega_{*T}}{\omega}\right)\left(1+\frac{\omega_D}{\omega}\right)$$

$$\frac{\delta n_e}{n} \simeq \frac{e\delta\phi}{T_e}\left(1-\frac{n_{eT}}{n}\left[1-\frac{\omega_{*e}(1+\eta_e)}{\omega}+\frac{\omega_{De}}{\omega}-\frac{\omega_{*e}(1+\eta_e)\omega_{De}}{\omega^2}\right]\right)$$

Fluid Dispersion Relation:

$$\begin{aligned}&\left(1-\frac{n_{eT}}{n}+\tau b_i\right)\\ &+\frac{\omega_{*e}}{\omega}\left(\frac{n_{eT}}{n}(1+\eta_e)+\frac{r_n}{R}(1-n_{eT}/n)+b_i(1+\eta_i)-1\right)\\ &+\frac{\omega_{*e}^2}{\omega^2}\left(\frac{n_{eT}}{n}\frac{r_n}{R}(1+\eta_e)+b_i(1+\eta_i)+\frac{1+\eta_i}{\tau}\frac{r_n}{R}\right)=0\end{aligned}$$

⇒ **Choose b_i to eliminate this term, leaving purely growing modes - gives wavelength for strongest modes (notice real frequency depends on b_i and η_i)**

For small, η_i fluid ion response not strictly valid...but...

$$\boxed{\gamma \simeq \omega_{*e}\sqrt{\frac{\frac{n_{eT}}{n}\frac{r_n}{R}(1+\eta_e)+b_i(1+\eta_i)+\frac{1+\eta_i}{\tau}\frac{r_n}{R}}{1-n_{eT}/n+\tau b_i}}}$$

$$\gamma_{TEM} \sim (k_\theta\rho_i)\sqrt{(n_T/n)g_i/r_n}$$

"Ubiquitous mode"

$$\gamma_{ITG} \sim (k_\theta\rho_i)\sqrt{\eta_i g_i/r_n}$$

Rayleigh-Taylor analog: $g = v_{th}^2/R$

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