Kinetic simulation of the VTF magnetic reconnection experiment

J. Egedal *, W. Fox, E. Belonohy, M. Porkolab

Massachusetts Institute of Technology, Plasma Science and Fusion Center, Cambridge, MA 02139, USA
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Abstract

A new numerical code is developed to assist the interpretation of experimental results from the Versatile Toroidal Facility (VTF) magnetic reconnection experiment. As input the code applies profiles of magnetic and electric fields consistent with experimental measurements. Applying Liouville’s theorem the code calculates the electron distributions functions. The moments of the distribution function provide the profiles of the electron current density, which are in agreement with the experimental observations. Thus the code provides new insight into the origin of the fast reconnection rates observed in VTF.

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1. Introduction

During magnetic reconnection, field lines in opposite directions cross link and form locally a magnetic cusp configuration. In low collisional plasmas, the reconnection process is observed on time scales much shorter than expected from theoretical considerations based upon resistive MHD models. The plasma dynamics in such a magnetic cusp configuration and the response to driven magnetic reconnection is investigated in the Versatile Toroidal Facility (VTF) experiment at the MIT Plasma Science and Fusion Center [1]. Three special features make the VTF experiment unique:

(i) the plasma production stage is separate from the reconnection drive;
(ii) the mean free path of the electrons is longer than the dimensions of the device; and
(iii) the reconnection rate observed in VTF is three orders of magnitude faster than the classical collisional value.

The understanding of the VTF experiments will help us to interpret reconnection phenomena in low collisional plasmas in space and in magnetic fusion devices.

The magnetic and electric fields throughout the poloidal cross-section are reconstructed experimentally during the reconnection process in VTF. The accurate characterization of these steady state profiles provides an excellent basis for detailed kinetic simulations of the reconnection process. With the known electric and magnetic fields Liouville’s equation is readily solved numerically, providing the detailed phase space distribution function of the electrons. These distributions reveal non-Maxwellian features, which are fundamental in accounting for the momentum balance of the electrons in the vicinity of the X-line. Besides an introduction to the ex-
experimental geometry, here we describe these recent kinetic simulations of the reconnection process in VTF.

2. The VTF experimental configuration

The poloidal cross section of VTF is shown in Fig. 1. The dashed lines represent contours of constant poloidal flux, which coincide with the poloidal projection of magnetic field lines. This magnetic cusp configuration is produced by poloidal field coils installed outside the stainless steel vacuum vessel. The solid lines represent contours of constant modulus of the poloidal magnetic field. Plasmas are produced by applying 2.45 GHz RF heating delivered by a 50 kW klystron amplifier via a ceramic window and a rectangular cross-section horn antenna. For the maximal operational cusp field the break down of the injected gas (normally hydrogen or argon at $10^{-5}$ Torr) occurs on the ECRH resonance surface as shown in Fig. 1. At this location the magnetic field strength is 87.5 mT and the electron cyclotron frequency is matched by the RF frequency. In addition to the cusp field, a toroidal field (0–200 mT) may be added using 18 toroidal field coils surrounding the vacuum vessel. Highly reproducible target plasmas are produced with pulse length up to 40 ms.

For the magnetic reconnection drive VTF is equipped with an ohmic coil system inducing toroidal electric field pulses (1 ms) of 10 V/m in the center of the vacuum vessel. The $E \times B$-drift associated with this electric field and the poloidal magnetic fields brings two opposite lobes of plasma together at the X-line. During this reconnection process the time evolution of the magnetic flux over the plasma cross-section is measured by a radial magnetic probe array. This movable probe has 20 pick-up coils for each of the three spatial directions, providing a spatial resolution of 1 cm in the radial direction. Due to the reproducibility of the VTF plasma, the spatial resolution in the vertical direction can be made arbitrarily high by moving the probe in between plasma shots. The plasma currents are calculated through $\mu_0 j = \nabla \times B$, assuming toroidal symmetry. The dynamic response of the plasma current was described in Ref. [2]. After an initial oscillation, the current density settles down to steady state with a diamond shaped cross-section. The current densities are typically three orders of magnitude less than the classical resistive value (given by $E/\eta_s$).

Direct measurements of the electrostatic potential are also conducted on VTF. An example of measured contours of the plasma potential is shown in Fig. 2. The diagram shows the plasma potential obtained by subtracting profiles, which are measured during periods with opposite sign of the induced electric field.

![Fig. 1. Poloidal cross-section of VTF. The solid contour lines represent the poloidal magnetic field strength. The dashed contour lines correspond to constant levels of the poloidal magnetic flux, $\Psi$, which coincide with magnetic field lines.](image1)

![Fig. 2. Change in plasma potential as a response to the induced reconnection electric field. The contours levels (labelled in units, V) are separated by 2 V.](image2)
3. Kinetic simulation approach

The mean free path between collisions for the electrons in VTF is in the order of 50 m. The thermal speed of the electrons is typically $10^6$ m/s, so during a particle confinement time, $\tau_e \sim 0.2$ ms, the electrons execute hundreds of complete bounce orbits in the poloidal cross-sections. It is therefore natural to seek the explanation for the observed plasma behavior in the properties of the particle orbits; in Ref. [2] it was found that the width of the diffusion region scales with the drift orbit width of the electrons. An example of a characteristic electron orbit is in Fig. 3.

A more detailed picture of the mechanisms responsible for the low current densities (and hereby fast reconnection) is obtained by solving Liouville’s equations, $df/dt = 0$, for the electron phase space density. This equation states that the distribution is constant along particle orbits through phase space $(x,v)$. Hence, we can equate the distribution function, $f$, for a point $(x_0,v_0)$ in the reconnection region to an isotropic distribution, $f_\infty$, in the ambient. This is done by following particle orbits back in time until they reach points, $(x_1,v_1)$, in the ambient. It then follows that

$$f(x_0,v_0) = f_\infty(v_1), \quad v_1 = |v_1|. \quad (1)$$

The electron distributions are therefore closely related to changes in kinetic energy that the particles undergo along their trajectories into the X-line region. This scheme for obtaining the phase space distribution has been applied earlier in reconnection configurations with no guide magnetic field (no magnetic field component along the X-line) and no electrostatic in-plane potential [3]. However, due to the requirement of quasi-neutrality, for the more generic reconnection configurations, in which a guide magnetic field is present, it is important also to include an in-plane electrostatic potential [4]. Because the structure of this potential is normally unknown the present numerical scheme had not been applied previously to configurations which do include a guide magnetic field. However, with our experimental knowledge of the electrostatic potential the scheme can be applied for the first time to configurations including a guide magnetic field.

Fig. 3 is solved by integrating the electron guiding center equations of motion in prescribed fields using a massively parallel computer code. Within the guiding center approximation, the magnetic moment, $\mu = mv^2/\langle 2B \rangle$ is assumed to be conserved and the velocity is fully specified by the velocity components parallel and perpendicular to the magnetic field, $(v_\parallel, v_\perp)$. A fourth order Runge–Kutta differential equation solver is used to integrate the following set of guiding center equations [5]:

$$\frac{dx}{dr} = -\frac{q}{mB^*}B^* \cdot \left( \frac{\mu}{q} \nabla B - E \right), \quad (2)$$

$$\frac{dx}{dt} = \frac{1}{B^*_y}(v_\parallel B^* + b \times \left( \frac{\mu}{q} \nabla B - E \right)), \quad (3)$$

where we use the notation

$$b = B/B, \quad B^* = B + \frac{mv_\parallel}{q} \nabla \times b, \quad (4)$$

$$B^*_y = b \cdot B^*. \quad (5)$$

Consistent with the experiment, the magnetic fields are approximated by the following expressions

$$B = \nabla \times A = b_0 (x\hat{x} - y\hat{y} + z\hat{z}). \quad (6)$$

The variable $b_0$ represent the gradient of the in-plane magnetic field. In VTF this variable can be varied in the interval, $b_0 = 0.09$ T. The magnetic configuration in Eq. (6) is currentless and is only a good approximation provided that $\mu_0 j \ll b_0$, where $j$ is the current density in the plasma. Experimentally we find $\mu_0 j < 0.01$ T/m, thus validating the currentless cusp approximations. Locally, the ratio between the cusp field and the guide field is given by $r/l_0$, where $r$ is the distance from the X-line. As apparent in Eq. (6) the guide field strength is given by $B_c = b_0 l_0$. Typically, $B_c \sim 0.09$ T in the VTF experiment.
The total electric field is given by
\[ E_z = -\nabla \Phi(x,y) + E_{zt}. \]

The second part of the potential \( \Phi \) is split into two separate parts:

\[ \Phi = \Phi_1 + \Phi_2, \]
\[ \Phi_1 = \frac{1}{4} E_z \delta^2 \log \left( \frac{x^2 + \delta^2}{y^2 + \delta^2} \right), \]
\[ \Phi_2 = P_1 \tanh(x/\gamma) + P_2 \tanh(y/\gamma). \]

In accordance with the experiment the part \( \Phi_1 \) ensures that \( \mathbf{E} \cdot \mathbf{B} = 0 \) for \( |xy| \gg \delta^2 \). Thus, the parameter \( \delta \) determines the size of the electron diffusion; the region where the plasma frozen-in condition, \( \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \), is violated and \( \mathbf{E} \cdot \mathbf{B} \) is finite. Experimentally we have found that \( \delta \approx \rho_e \) is a good approximation, where \( \rho_e = \sqrt{mv/qb_0} \) is the drift orbit width of the electrons [2].

The second part of the potential, \( \Phi_2 \), adds the possibility of including an electric field component across the X-line. Even a modest electric field component of this type effectively eliminates singularities in the orbit dynamics at the X-line. Typical values used are \( \gamma = \delta \) and \( P_1 = P_2 \equiv T_e/5 \).

In VTF the plasma potential (compared to the vacuum vessel walls) is typically \( V_p \approx 2T_e \). This potential confines the electrons electrostatically and ensures that the electrons and the ions are lost at the same rate and quasi neutrality is maintained. Hence, in addition to \( \Phi_1 \) and \( \Phi_2 \) we also included an electric potential which reflects the electrons when they reach a distance of 1 m from the X-line.

A useful quantity for understanding the orbit dynamics during reconnection is the generalized particle momentum in the z-direction

\[ p_z = q b_0 x y - q E_z t + m v_{||} \frac{B_z}{B}. \]

This quantity, derived from the guiding center Lagrangian, is a constant of motion, as the linear cusp is independent of the z-coordinate [4]. Because \( p_z \) is a constant of motion it follows from the expressions (7) and (10) that the turning points (where \( v_{||} = 0 \)) of the particles are frozen to the magnetic flux function \( A_z \). Even as the field line drifts in time, which is the case when \( E_z \neq 0 \), the particle still follows the field line. Note that because the electrostatic potential \( \Phi \) does not appear in the expression for \( p_z \), this result is independent of \( \Phi \). In our scheme for solving Eq. (1) we assume that plasma with density, \( n_1 \), is Maxwellian, \( f_\infty(v_{||}, v_{\perp}) = n_1(m_e/(2\pi T_e))^{3/2} \exp(-m_e(v_{\perp}^2 + v_{||}^2)/(2T_e)) \), at a given location, \( x_1 \), outside the reconnection region (we typically use \( (x_1, y_1) = (10(\delta, \delta)) \)). In order to obtain \( f \) at a location \( (x_0, y_0) \) we apply Eq. (10) to calculate the time, \( \Delta t \), it takes an electron to travel from \( (x_1, y_1) \) to \( (x_0, y_0) \). Thus, we find that

\[ q E_z \Delta t = q b_0 (x_0 y_0 - x_1 y_1) + m v_{||0} B_z \]

\[ \approx \gamma \sqrt{v_{||0}^2 + v_{\perp0}^2}. \]

In the simulations a 40 x 40 grid is applied for \( (x_0, y_0) \). The points are selected unevenly with the highest spatial resolution in the X-line region. For each

Fig. 4. Contour of the electron velocity distribution for a location close to the X-line.
point in the spatial grid of $80 \times 40$ points is applied for $(v_\parallel_0, v_{\perp 0})$. In total each simulation includes the characterization of $\sim 5 \cdot 10^6$ guiding center trajectories through the reconnection region.

4. Preliminary results

An example of the distribution function evaluated close to the X-line is shown in Fig. 4. As seen, the distribution contains strong non-Maxwellian features. Through associated terms in the electrons pressure tensor, these non-Maxwellian features are expected to be important for the momentum balance of the electrons throughout the reconnection region.

The current profile is obtained from the first moment of the distribution function; an example is shown in Fig. 5. The numerical current density profiles are in good agreement with the experimental observations. Further investigations are underway to benchmark the numerical results against experimental data for a range of experimental parameters including, $E_z$, $l_0$ and $b_0$.

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References